

Controllable coupling and quantum correlation dynamics of two double quantum dots coupled via a transmission line resonator

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We propose a theoretical scheme to generate a controllable and switchable coupling between two double-quantum-dot (DQD) spin qubits by using a transmission line resonator (TLR) as a bus system. We study dynamical behaviors of quantum correlations described by entanglement correlation (EC) and discord correlation (DC) between two DQD spin qubits when the two spin qubits and the TLR are initially prepared in X -type quantum states and a coherent state, respectively. We demonstrate that in the EC death regions there exist DC stationary states in which the stable DC amplification or degradation can be generated during the dynamical evolution. It is shown that these DC stationary states can be controlled by initial-state parameters, the coupling, and detuning between qubits and the TLR. We reveal the full synchronization and anti-synchronization phenomena in the EC and DC time evolution, and show that the EC and DC synchronization and anti-synchronization depends on the initial-state parameters of the two DQD spin qubits. These results shed new light on dynamics of quantum correlations.

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I. INTRODUCTION

It is well known that quantum entanglement [1, 2] and quantum discord [3] are two different types of quantum correlations. In recent years, it has been widely recognized that both of the two quantum correlations are essential quantum resources which can be used to realize quantum information processing, and the fact that quantum discord is a more general concept to measure quantum correlations than quantum entanglement since there is a nonzero quantum discord in some separable mixed states [4–11]. However, the question of the relation between entanglement correlation (EC) and discord correlation (DC) is still an open problem in the field of quantum correlations.

Interactions of quantum systems are at the core of quantum information processing. In particular, quantum computing requires that inter-qubit interactions are controllable, and can be selectively switched on and off [12, 13]. Many physical systems have been explored for the realization of practical quantum information processors, such as cavity quantum electrodynamics (QED) system [14], optical system [15], and solid-state system [16–22]. Solid-state devices are the promising candidates for the implementation of quantum computation due to the possibility of fabricating large integrated networks. Among different kinds of solid systems, double quantum dot (DQD) system [23–31] and a transmission line resonator (TLR) [32–38] are particularly attractive because of the relative long spin coherence time and high controllability of DQD system and quantum bus function of the TLR. Several proposals have been proposed to realize controllable couplings and local operations of quantum dot (QD) qubits via

a TLR [39–42]. In Ref. [39], the electron spins in nanowire QDs couple to the electric component of the resonator electromagnetic field and enable quantum information processing in an all-electrical fashion. In Ref. [40], all-electrical coupling between QD spin qubits and a TLR are also used to produce effective interactions between spin qubits. In Ref. [41], two QD spin qubits were embedded in a superconducting microstrip cavity, virtual photons in a common cavity mode could mediate coherent interactions between two distant qubits. In this paper, we want to propose a new scheme to implement the controllable coupling between double-quantum-dot (DQD) spin qubits in terms of the magnetic coupling between DQDs and the TLR by using the TLR as the quantum bus system. In our scheme, each DQD spin qubit couples to the TLR through the magnetic field generated by the current of the TLR. We will study the relation between EC and DC by investigating the dynamic behaviors of EC and DC between the two DQD spin qubits in the combining system consisting of two DQDs and a TLR. We will show the appearance of the DC stationary states in the EC death regimes and demonstrate the full synchronization and anti-synchronization of DC and EC in the time evolution. In particular, it shall be indicated that the DC between the two DQD spin qubits can reach and keep its maximum fixed even if the EC disappears completely in the time evolution for certain initially prepared X -type states.

This paper is organized as follows. In Sec. II, we present our coupling scheme of two DQD spin qubits by using the TLR as the quantum bus system, and show that the inter-spin-qubit interaction in our coupling scheme is controllable and switchable. In Sec. III, we investigate dynamical behaviors of EC and DC between two DQD qubits in the controllable coupling scheme, and discuss the relation between EC and DC for X -type initial states. The stationary amplification and degradation of the DC and the time synchronization and anti-synchronization of the EC and DC are revealed in the time evolution of the combined hybrid system. Finally, we con-

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clude this work in Sec. IV.

II. CONTROLLABLE COUPLING SCHEME OF TWO DQD SPIN QUBITS

In this section, we propose a scheme to generate an effective controllable interaction between two DQD spin qubits by using a TLR as the data bus. The combined system under our consideration is indicated in Fig. 1. It consists of two DQDs charged with two excess electrons and a TLR. The length of the TLR is L . The two DQDs, 1 and 2, are placed at the positions $L/4$ and $3L/4$ of the TLR, respectively. These are the antinode of the quantized current of the TLR. Two electron spins in each DQD are localized in adjacent QDs, coupled via tunnelling. The distance between the lower dot and the TLR and the distance between the two dots are r .

In terms of the annihilation and creation operators a and a^\dagger , we can write the Hamiltonian for the TLR as

$$\hat{H}_r = \hbar\omega_r \hat{a}^\dagger \hat{a}, \quad (1)$$

where ω_r is the frequency of the TLR. The Hamiltonian of a DQD is most conveniently written in the two-electron singlet-triplet basis $\{|S_{11}\rangle, |T_{11}^0\rangle, |T_{11}^+\rangle, |T_{11}^-\rangle, |S_{02}\rangle\}$ with the quantization axis in the z -direction as [42–45]

$$\begin{aligned} \hat{H}_q = & E_S |S_{11}\rangle\langle S_{11}| + (\Delta_0 + E_S) |S_{02}\rangle\langle S_{02}| \\ & + g_B \mu_B B_e (|T_{11}^+\rangle\langle T_{11}^+| - |T_{11}^-\rangle\langle T_{11}^-|) \\ & + E_T |T_{11}^0\rangle\langle T_{11}^0| + t(|S_{11}\rangle\langle S_{02}| + |S_{02}\rangle\langle S_{11}|), \end{aligned} \quad (2)$$

where the two-electron singlet-triplet basis are given by

$$\begin{aligned} |T_{11}^+\rangle &= |\uparrow\uparrow\rangle, \quad |T_{11}^0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \quad |T_{11}^-\rangle = |\downarrow\downarrow\rangle, \\ |S_{11}\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \end{aligned} \quad (3)$$

and the auxiliary singlet state with two electrons in one quantum dot, $|S_{02}\rangle$, is coupled via tunneling t to the separated singlet, $|S_{11}\rangle$, where the subscript m, n denotes the dot occupancy. E_S and E_T are the energy of the $|S_{11}\rangle$ and $|T_{11}^0\rangle$ states, respectively. Δ_0 is the energy difference between the $|S_{11}\rangle$ and $|S_{02}\rangle$ states set by the electric field. B_e is the external magnetic field, μ_B is the Bohr magneton and g_B the electron spin g -factor.

If the external magnetic field in the upper dot B_e^u is different from that in the lower dot B_e^l , and if we consider the effect of the hyperfine interaction with the nuclear spins which can be studied by adding a static frozen effective nuclear field B_N^u (B_N^l) at the upper (lower) dot to the total magnetic field [43–45], the interaction between the spin and the total magnetic field in each DQD is given by $\hat{H}_I = -g_B \mu_B (\hat{B}_u \cdot \hat{S}_u + \hat{B}_l \cdot \hat{S}_l)/\hbar$ which can be rewritten as

$$\begin{aligned} \hat{H}_I = & -\frac{g_B \mu_B}{\hbar} (\hat{B}_u + \hat{B}_l) \cdot (\hat{S}_u + \hat{S}_l)/2, \\ & -\frac{g_B \mu_B}{\hbar} (\hat{B}_u - \hat{B}_l) \cdot (\hat{S}_u - \hat{S}_l)/2, \end{aligned} \quad (4)$$

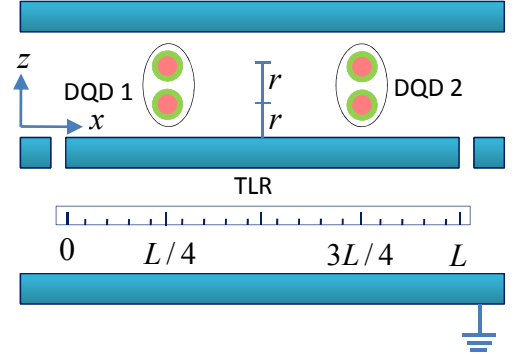


FIG. 1: The proposed setup with two DQDs, biased with external potential Δ_0^j , $j = 1, 2$, magnetically coupled to a TLR of length L . The two DQDs 1 and 2 are placed at the positions $L/4$ and $3L/4$ of the TLR, respectively. These are the antinode of the quantized current of the TLR. In each DQD, both of the distance between the lower dot and the TLR and the distance between the two dots are r .

where the total magnetic fields which an electron in each DQD experiences are $\hat{B}_u = \hat{B}_N^u + \hat{B}_e^u$ and $\hat{B}_l = \hat{B}_N^l + \hat{B}_e^l$, the sum operators and the difference operators of the two electron spins can be expressed in terms of the two-electron singlet-triplet basis as

$$\begin{aligned} \hat{S}_u^x + \hat{S}_l^x &= \frac{\hbar}{\sqrt{2}}(|T_{11}^0\rangle\langle T_{11}^-| + |T_{11}^0\rangle\langle T_{11}^+| + \text{H.c.}), \\ \hat{S}_u^y + \hat{S}_l^y &= \frac{\hbar}{\sqrt{2}}(i|T_{11}^0\rangle\langle T_{11}^-| - i|T_{11}^0\rangle\langle T_{11}^+| + \text{H.c.}), \\ \hat{S}_u^z + \hat{S}_l^z &= \hbar(|T_{11}^+\rangle\langle T_{11}^+| - |T_{11}^-\rangle\langle T_{11}^-|), \\ \hat{S}_u^x - \hat{S}_l^x &= \frac{\hbar}{\sqrt{2}}(|S_{11}\rangle\langle T_{11}^-| - |S_{11}\rangle\langle T_{11}^+| + \text{H.c.}), \\ \hat{S}_u^y - \hat{S}_l^y &= \frac{\hbar}{\sqrt{2}}(i|S_{11}\rangle\langle T_{11}^-| + i|S_{11}\rangle\langle T_{11}^+| + \text{H.c.}), \\ \hat{S}_u^z - \hat{S}_l^z &= \hbar(|S_{11}\rangle\langle T_{11}^0| + |T_{11}^0\rangle\langle S_{11}|). \end{aligned} \quad (5)$$

Eqs. (4) and (5) indicate that the homogeneous part of the magnetic field $\hat{B}_u + \hat{B}_l$ simply adds vectorially to the external field B_e , changing slightly the Zeeman splitting and preferred spin orientation of the triplet states. The inhomogeneous part $\Delta\hat{B} = \hat{B}_u - \hat{B}_l$, on the other hand, couples the triplet states $\{|T_{11}^0\rangle, |T_{11}^+\rangle, |T_{11}^-\rangle\}$ to the singlet state $|S_{11}\rangle$.

The degree of mixing between two states will depend strongly on the energy difference between them. In the case of $g_B \mu_B B_e \gg t$ and $g_B \mu_B \sqrt{\langle \Delta B^2 \rangle}$, the spin-aligned states $|T_{11}^+\rangle$ and $|T_{11}^-\rangle$ are split off due to Zeeman energy $g_B \mu_B B_e$, then the perturbation of these states will be small, while the spin-anti-aligned state $|T_{11}^0\rangle$ remains mixed with the state $|S_{11}\rangle$. Then we can write the Hamiltonian of this combined system as

$$\begin{aligned} \hat{H} = & \hbar\omega_r \hat{a}^\dagger \hat{a} + \hbar \sum_{j=1}^2 \frac{\omega_j}{2} \hat{\sigma}_z^j + (\Delta_0^j + E_S^j) |S_{02}\rangle_j \langle S_{02}| \\ & + \frac{1}{2} g_B \mu_B (\Delta\hat{B}_j^z) \hat{\sigma}_x^j + t_j (|S_{11}\rangle_j \langle S_{02}| + |S_{02}\rangle_j \langle S_{11}|), \end{aligned} \quad (6)$$

where $\omega_j = (E_S^j - E_T^j)/\hbar$, $\Delta\hat{B}_j^z$ is the z -component of $\Delta\hat{B}_j$, and we have introduced Pauli spin operators

$$\begin{aligned}\sigma_z^j &= |S_{11}\rangle_j\langle S_{11}| - |T_{11}^0\rangle_j\langle T_{11}^0|, \\ \sigma_x^j &= |S_{11}\rangle_j\langle T_{11}^0| + |T_{11}^0\rangle_j\langle S_{11}|.\end{aligned}\quad (7)$$

In Hamiltonian (6) we have used only $|T_{11}^0\rangle_j$ and $|S_{11}\rangle_j$ as a spin-qubit degree of freedom. Qubit-TLR Interaction is induced naturally by the magnetic field gradient $\Delta\hat{B}_j^z$ which includes the contributions from the nuclear magnetic field and the TLR itself

$$\Delta\hat{B}_j^z = \Delta\hat{B}_{N_j}^z + \frac{\mu_0\hat{I}_j}{4\pi r}, \quad (8)$$

where μ_0 is the vacuum permeability, and $\Delta\hat{B}_{N_j}^z$ is the gradient of the longitudinal component of the nuclear magnetic fields between the two dots of the j th DQD. The current \hat{I}_j at the position of the j th DQD due to the resonator of length L is quantized as

$$\hat{I}_j = (-1)^{j-1} \sqrt{\frac{\hbar\omega_r}{Ll}} (\hat{a} + \hat{a}^\dagger), \quad (9)$$

where l is the inductance per unit length of the TLR.

Substituting Eqs. (8) and (9) into Eq. (6) we arrive at the following Hamiltonian

$$\begin{aligned}\hat{H} &= \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar\sum_{j=1}^2\frac{\omega_j}{2}\hat{\sigma}_z^j + (\Delta_0^j + E_S^j)|S_{02}\rangle_j\langle S_{02}| \\ &+ \frac{g_B\mu_B}{2}\left[\Delta\hat{B}_{N_j}^z + (-1)^{j-1}\frac{\mu_0}{4\pi r}\sqrt{\frac{\hbar\omega_r}{Ll}}(\hat{a} + \hat{a}^\dagger)\right]\hat{\sigma}_x^j \\ &+ t_j(|S_{11}\rangle_j\langle S_{02}| + |S_{02}\rangle_j\langle S_{11}|).\end{aligned}\quad (10)$$

In the interaction picture with respect to the term $(\Delta_0^j + E_S^j)|S_{02}\rangle_j\langle S_{02}|$, we obtain the interaction Hamiltonian after discarding rapidly oscillating terms

$$\begin{aligned}\hat{H}_1 &= \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar\sum_{j=1}^2\frac{\omega_j}{2}\hat{\sigma}_z^j \\ &+ \frac{g_B\mu_B}{2}\left[\Delta\hat{B}_{N_j}^z + (-1)^{j-1}\frac{\mu_0}{4\pi r}\sqrt{\frac{\hbar\omega_r}{Ll}}(\hat{a} + \hat{a}^\dagger)\right]\hat{\sigma}_x^j.\end{aligned}\quad (11)$$

When $\omega_j \gg g_B\mu_B\Delta\hat{B}_{N_j}^z/(2\hbar)$, and $\omega_j + \omega_r \gg \omega_j - \omega_r$, $\frac{g_B\mu_B\mu_0}{8\hbar\pi r}\sqrt{\frac{\hbar\omega_r}{Ll}}$, the rotating-wave approximation can be applied to get an effective Hamiltonian of the combined system

$$\hat{H}_2 = \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar\sum_{j=1}^2\frac{\omega_j}{2}\hat{\sigma}_z^j + (-1)^{j-1}\hbar g(a\hat{\sigma}_+^j + \hat{\sigma}_-^ja^\dagger), \quad (12)$$

where the effective coupling constant $g = \frac{g_B\mu_B\mu_0}{8\hbar\pi r}\sqrt{\frac{\hbar\omega_r}{Ll}}$. Eq. (12) is the Hamiltonian of the usual Jaynes-Cummings model of two atoms with $\hat{\sigma}_+^j = |S_{11}\rangle_j\langle T_{11}^0|$ and $\hat{\sigma}_-^j = |T_{11}^0\rangle_j\langle S_{11}|$.

If both DQDs are strongly detuned from the TLR, i.e., $|\delta_j| = |\omega_j - \omega_r| \gg |g|$, we can adiabatically eliminate the TLR mode using the following transformation

$$\hat{U} = \exp\left[\frac{g}{\delta_1}(\hat{a}^\dagger\hat{\sigma}_1^- - \hat{a}\hat{\sigma}_1^+) - \frac{g}{\delta_2}(\hat{a}^\dagger\hat{\sigma}_2^- - \hat{a}\hat{\sigma}_2^+)\right]. \quad (13)$$

To second order in the small parameters g/δ_j , the effective Hamiltonian becomes

$$\begin{aligned}\hat{H}_{\text{eff}} &= \hbar\omega_r\hat{a}^\dagger\hat{a} + \frac{\hbar}{2}\sum_{j=1}^2\left[\omega_j + 2\frac{g^2}{\delta_j}\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)\right]\hat{\sigma}_z^j \\ &- \frac{\hbar g^2(\delta_1 + \delta_2)}{2\delta_1\delta_2}(\hat{\sigma}_+^1\hat{\sigma}_-^2 + \hat{\sigma}_-^1\hat{\sigma}_+^2),\end{aligned}\quad (14)$$

where the last term describes swap of the DQD states through virtual interaction with the TLR.

To easily switch on and off the inter-qubit coupling is one of the most important open problems in quantum computing hardware. Here we propose a way to overcome the severe problem. From Eqs. (8), (12) and (14) we can see that the inter-spin-qubit coupling between the two DQDs is mediated by the TLR, and the interaction between each DQD and the TLR originated from the magnetic field gradient $\mu_0 I_j/4\pi r$ between the two dots of each DQD, which is produced by the TLR. So if additionally we add an asymmetric magnetic field B_z in the z -direction, then we have

$$\Delta B_j^z = \Delta B_{N_j}^z + \frac{\mu_0 I_j}{4\pi r} + dB_z^j, \quad (15)$$

where dB_z^j is the difference of the asymmetric magnetic field B_z between the two QDs of the j th DQD. Then by tuning the magnitude of dB_z^j , we can get $\Delta B_j^z = 0$. This implies that the interaction between the j th DQD and the TLR is turned off. Therefore, we can conclude that the effective coupling between the two DQDs in the present spin-qubit coupling scheme is controllable and switchable.

III. DYNAMICS OF QUANTUM CORRELATIONS BETWEEN TWO DQD SPIN QUBITS

In this section, we investigate dynamics of quantum correlations between two DQD spin qubits. We will study dynamic evolution of quantum entanglement and quantum discord when the two DQD spin qubits are initially in three-parameter two-qubit X -type quantum states [46] which play an important role in a number of physical systems.

Firstly, we solve the Hamiltonian of the DQD-TLR system under our consideration. In the interaction picture with respect to the first term $\hbar\omega_r\hat{a}^\dagger\hat{a}$ of the Hamiltonian \hat{H}_{eff} (14), we get

$$\hat{\mathcal{H}} = \hbar\sum_{j=1}^2\Omega_j\hat{\sigma}_z^j - \hbar\chi(\hat{\sigma}_+^1\hat{\sigma}_-^2 + \hat{\sigma}_-^1\hat{\sigma}_+^2). \quad (16)$$

where the inter-qubit and qubit-TLR couplings are given by

$$\Omega_j = \frac{1}{2}\left[\omega_j + 2\frac{g^2}{\delta_j}\left(\hat{N} + \frac{1}{2}\right)\right], \quad \chi = \frac{g^2(\delta_1 + \delta_2)}{2\delta_1\delta_2}, \quad (17)$$

with $\hat{N} = a^\dagger a$ being the number operator of the TLR. We can write $\hat{\mathcal{H}}$ in the basis $\{|S\rangle_1|S\rangle_2, |S\rangle_1|T\rangle_2, |T\rangle_1|S\rangle_2, |T\rangle_1|T\rangle_2\}$ as

$$\hat{\mathcal{H}} = \hbar \begin{pmatrix} \Omega_1 + \Omega_2 & 0 & 0 & 0 \\ 0 & \Omega_1 - \Omega_2 & -\chi & 0 \\ 0 & -\chi & -\Omega_1 + \Omega_2 & 0 \\ 0 & 0 & 0 & -\Omega_1 - \Omega_2 \end{pmatrix}. \quad (18)$$

Here and in the after we use $|S\rangle_j$ and $|T\rangle_j$ to replace $|S_{11}\rangle_j$ and $|T_{11}^0\rangle_j$ in Sec. II, respectively. The four eigenstates of the Hamiltonian $\hat{\mathcal{H}}$ can be obtained as

$$\begin{aligned} |\Psi_1\rangle &= |S\rangle_1|S\rangle_2, \\ |\Psi_2\rangle &= \cos(\theta/2)|S\rangle_1|T\rangle_2 + \sin(\theta/2)|T\rangle_1|S\rangle_2, \\ |\Psi_3\rangle &= -\sin(\theta/2)|S\rangle_1|T\rangle_2 + \cos(\theta/2)|T\rangle_1|S\rangle_2, \\ |\Psi_4\rangle &= |T\rangle_1|T\rangle_2, \end{aligned} \quad (19)$$

with the corresponding eigenvalues

$$\begin{aligned} E_1 &= -E_4 = \hbar(\Omega_1 + \Omega_2), \\ E_2 &= -E_3 = \hbar\sqrt{(\Omega_1 - \Omega_2)^2 + \chi^2}. \end{aligned} \quad (20)$$

The mixing angle in Eq. (19) is defined by

$$\sin \theta = -\hbar\chi/E_2, \quad \cos \theta = \hbar(\Omega_1 - \Omega_2)/E_2. \quad (21)$$

From Eqs. (19) and (20) we get the time evolution operator of $\hat{\mathcal{H}}$ as $\hat{U} = \sum_{n=1}^4 \exp(-iE_n t/\hbar) |\Psi_n\rangle \langle \Psi_n|$ which can be expressed as

$$\hat{U} = \begin{pmatrix} e^{-iE_1 t/\hbar} & 0 & 0 & 0 \\ 0 & e^{-iE_2 t/\hbar} - \eta & \kappa & 0 \\ 0 & \kappa & e^{-iE_3 t/\hbar} + \eta & 0 \\ 0 & 0 & 0 & e^{-iE_4 t/\hbar} \end{pmatrix} \quad (22)$$

with the following parameters

$$\begin{aligned} \kappa &= (e^{-iE_2 t/\hbar} - e^{-iE_3 t/\hbar}) \sin(\theta/2) \cos(\theta/2), \\ \eta &= (e^{-iE_2 t/\hbar} - e^{-iE_3 t/\hbar}) \sin^2(\theta/2). \end{aligned} \quad (23)$$

We assume the two DQDs are initially prepared in a class of state with maximally mixed marginals ($\hat{\rho}_{A(B)} = \hat{I}_{A(B)}/2$) described by the three-parameter X-type density matrix $\hat{\rho}(0) = (\hat{I}_{AB} + \sum_{i=1}^3 c_i \hat{\sigma}_A^i \otimes \hat{\sigma}_B^i)/4$ which can be expressed as

$$\hat{\rho}(0) = \frac{1}{4} \begin{pmatrix} 1+c_3 & 0 & 0 & c_1-c_2 \\ 0 & 1-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1-c_3 & 0 \\ c_1-c_2 & 0 & 0 & 1+c_3 \end{pmatrix}, \quad (24)$$

where c_i ($0 \leq |c_i| \leq 1$) are real numbers satisfying the unit trace and positivity conditions of the density operator $\hat{\rho}$. The density operator $\hat{\rho}$ includes the Werner states and the Bell states as two special cases. Here and in the after we use the label $A(B)$ to denote the first (second) DQD.

If we initially prepare the TLR in the coherent state

$$|\psi_r(0)\rangle = |\alpha\rangle_r = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (25)$$

Then, the reduced density matrix of the two DQDs at time t can be obtained as

$$\hat{\rho}(t) = \text{Tr}_r(\hat{U}^\dagger \hat{\rho}(0) \otimes |\psi_r(0)\rangle \langle \psi_r(0)| \hat{U}). \quad (26)$$

For simplicity but without loss generality, here we consider the case of two identical DQDs, i.e., we take $\omega_A = \omega_B = \omega$ and $\delta_A = \delta_B = \delta$. Then from Eqs. (17) and (21) we have $\cos \theta = 0$. In this case, the explicit form of density operator at time t is

$$\hat{\rho}(t) = \frac{1}{4} \begin{pmatrix} 1+c_3 & 0 & 0 & c_0 \\ 0 & 1-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1-c_3 & 0 \\ c_0^* & 0 & 0 & 1+c_3 \end{pmatrix}, \quad (27)$$

where we have introduced the following parameter

$$c_0 = (c_1 - c_2) e^{2i(\omega+g^2/\delta)t} \exp[-|\alpha|^2 (1 - e^{4ig^2t/\delta})]. \quad (28)$$

In what follows, we study dynamic properties of quantum correlations between the two DQD spin qubits in terms of the expression of the density operator of the two DQD spin qubits given in Eq. (27). We investigate in detail EC and DC described by concurrence [47] and quantum discord, respectively. When the density matrix of the two qubit system has an X-type structure, the concurrence has a simple analytic expression [46]

$$C(t)_{AB} = 2 \max\{0, \Lambda_1(t), \Lambda_2(t)\}, \quad (29)$$

where $\Lambda_1(t) = |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}$ and $\Lambda_2(t) = |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}$ with ρ_{ij} ($i, j = 1, 2, 3, 4$) being the matrix elements of the density operator $\hat{\rho}(t)$ in Eq. (27). For the density operator given by Eq. (27) we have

$$\begin{aligned} \Lambda_1(t) &= \frac{1}{4} (|c_0| - |1 - c_3|), \\ \Lambda_2(t) &= \frac{1}{4} (|c_1 + c_2| - |1 + c_3|). \end{aligned} \quad (30)$$

Now we turn to investigate dynamic evolution of quantum discord correlation between two DQD qubits under our consideration. Quantum discord [3] is defined as the difference between the total correlation and the classical correlation with the following expression

$$\mathcal{D}(\hat{\rho}) = \mathcal{I}(\hat{\rho}_A : \hat{\rho}_B) - C(\hat{\rho}). \quad (31)$$

Here the total correlation in a bipartite quantum state $\hat{\rho}$ is measured by quantum mutual information given by

$$\mathcal{I}(\hat{\rho}_A : \hat{\rho}_B) = S(\hat{\rho}_A) + S(\hat{\rho}_B) - S(\hat{\rho}), \quad (32)$$

where $S(\hat{\rho}) = -\text{Tr}(\hat{\rho} \log \hat{\rho})$ is the von Neumann entropy, $\hat{\rho}_A = \text{Tr}_B(\hat{\rho})$ and $\hat{\rho}_B = \text{Tr}_A(\hat{\rho})$ are the reduced density operators for subsystems A and B , respectively. And the classical correlation between the two subsystems A and B can be defined as

$$\begin{aligned} C(\hat{\rho}) &= \max_{\{\hat{P}_k\}} \left[S(\hat{\rho}_A) - \sum_k p_k S(\hat{\rho}_A^{(k)}) \right] \\ &= S(\hat{\rho}_A) - \min_{\{\hat{P}_k\}} \left[\sum_k p_k S(\hat{\rho}_A^{(k)}) \right]. \end{aligned} \quad (33)$$

Here $\{\hat{P}_k\}$ is a set of projects performed locally on the subsystem B , and $\hat{\rho}_A^{(k)} = \frac{1}{p_k} \text{Tr}_B[(\hat{I}_A \otimes \hat{P}_k) \hat{\rho} (\hat{I}_A \otimes \hat{P}_k)]$ is the state of the subsystem A conditioned on the measurement outcome labeled by k , where $p_k = \text{Tr}_{AB}[(\hat{I}_A \otimes \hat{P}_k) \hat{\rho} (\hat{I}_A \otimes \hat{P}_k)]$ denotes the probability relating to the outcome k , and \hat{I}_A denotes the identity operator for the subsystem A .

In order to obtain the quantum discord for the two qubit-system [48], we first evaluate the mutual information of state $\hat{\rho}(t)$ given in Eq. (27). The four eigenvalues of $\hat{\rho}(t)$ are

$$\begin{aligned} \lambda_{1,2} &= \frac{1}{4}(1 - c_3 \pm |c_1 + c_2|), \\ \lambda_{3,4} &= \frac{1}{4}(1 + c_3 \pm |c_0|). \end{aligned} \quad (34)$$

Then the mutual information reads

$$\mathcal{I}(\hat{\rho}_A : \hat{\rho}_B) = 2 + \sum_{i=1}^4 \lambda_i \log_2 \lambda_i. \quad (35)$$

Note that here we have used $S(\hat{\rho}_A(t)) = S(\hat{\rho}_B(t)) = 1$, since the two reduced density matrixes $\hat{\rho}_A(t)$ and $\hat{\rho}_B(t)$ are maximally mixed, that is $\hat{\rho}_A(t) = \hat{I}_A/2$ and $\hat{\rho}_B(t) = \hat{I}_B/2$.

For calculation of the amount for the classical correlation $C(\hat{\rho})$ defined in Eq. (33), we propose the complete set of orthogonal projectors $\{\hat{P}_k = |\theta_k\rangle\langle\theta_k|, k = \parallel, \perp\}$ for a local measurement performed on the subsystem B , where the two projectors are defined in terms of the following two orthogonal states

$$\begin{aligned} |\theta_{\parallel}\rangle &= \cos \varphi |0\rangle + e^{i\phi} \sin \varphi |1\rangle, \\ |\theta_{\perp}\rangle &= e^{-i\phi} \sin \varphi |0\rangle - \cos \varphi |1\rangle, \end{aligned} \quad (36)$$

with $0 \leq \varphi \leq \pi/2$ and $0 \leq \phi \leq 2\pi$. After the two project measurements with $p_{\parallel} = p_{\perp} = 1/2$, the reduced density matrixes of subsystem A read as

$$\begin{aligned} \hat{\rho}_A^{\parallel} &= \frac{1}{4} \begin{pmatrix} 2(1 + c_3 \cos 2\varphi) & \epsilon \sin 2\varphi \\ \epsilon^* \sin 2\varphi & 2(1 - c_3 \cos 2\varphi) \end{pmatrix}, \\ \hat{\rho}_A^{\perp} &= \frac{1}{4} \begin{pmatrix} 2(1 - c_3 \cos 2\varphi) & -\epsilon \sin 2\varphi \\ -\epsilon^* \sin 2\varphi & 2(1 + c_3 \cos 2\varphi) \end{pmatrix}, \end{aligned} \quad (37)$$

where we have introduced the parameter

$$\epsilon = (c_1 + c_2)e^{-i\phi} + c_0 e^{i\phi}. \quad (38)$$

According to Eq. (37), it is straightforward to obtain the eigenvalues of the reduced density matrix $\hat{\rho}_A^k$ as follows

$$\zeta_{\parallel}^{1,2} = \zeta_{\perp}^{1,2} = \frac{1}{2}(1 \pm \Gamma), \quad (39)$$

where we have defined Γ as

$$\Gamma = \sqrt{c_3^2 \cos^2(2\varphi) + \frac{|\epsilon|^2}{4} \sin^2(2\varphi)}, \quad (40)$$

then we have

$$\begin{aligned} S(\rho_A^{\parallel}) &= S(\rho_A^{\perp}) = f(\Gamma) \\ &= -\frac{1+\Gamma}{2} \log_2 \left(\frac{1+\Gamma}{2} \right) - \frac{1-\Gamma}{2} \log_2 \left(\frac{1-\Gamma}{2} \right) \end{aligned} \quad (41)$$

which leads to the classical correlation

$$C(\hat{\rho}(t)) = 1 - \min_{\varphi, \phi} [f(\Gamma)]. \quad (42)$$

Since the function $f(\Gamma)$ is a monotonically decreasing function, in order to get the minimal value of $f(\Gamma)$ we should choose proper parameters φ and ϕ to ensure the parameter Γ defined in Eq. (40) is maximal. From Eqs. (38) and (40) it is easy to get the following inequality

$$\begin{aligned} \Gamma &\leq \sqrt{c_3^2 \cos^2(2\varphi) + c_4^2 \sin^2(2\varphi)} \\ &\leq \begin{cases} |c_3|, & \text{for } |c_3| > c_4, \\ c_4, & \text{for } |c_3| < c_4. \end{cases} \end{aligned} \quad (43)$$

where we have introduced the parameter

$$c_4 = \frac{|c_1 + c_2| + |c_0|}{2}. \quad (44)$$

If we define $\gamma(t)$ as

$$\gamma(t) = \max[|c_3|, c_4], \quad (45)$$

then the classical correlation can be expressed as

$$C(\hat{\rho}(t)) = \sum_{m=1}^2 \frac{1 + (-1)^m \gamma}{2} \log_2 [1 + (-1)^m \gamma]. \quad (46)$$

Therefore, the quantum discord can be written as

$$\mathcal{D}(\hat{\rho}(t)) = 2 + \sum_{i=1}^4 \lambda_i \log_2 \lambda_i - C(\hat{\rho}(t)), \quad (47)$$

where the amount of the classical correlation $C(\hat{\rho}(t))$ is given by Eq. (46). In principle, we have obtained the dynamics of the quantum discord according the above expression given in Eq. (47), provided that we know the initial condition of the system. In what follows we will study dynamic properties of the quantum entanglement and discord for some initial states in detail.

From the density operator of the two DQD spin qubits at time t (27) and Eq. (45) we can obtain the parameter

$$\gamma(t) = \begin{cases} |c_3|, & \text{for } 2|c_3| > |c_1 - c_2| + |c_1 + c_2|, \\ c_4, & \text{for } 2|c_3| < |c_1 - c_2|e^{-2|\alpha|^2} + |c_1 + c_2|. \end{cases} \quad (48)$$

which indicates that the classical correlation expressed by Eq. (46) largely depends on the initial state parameters of the two DQDs and the TLR $\{c_1, c_2, c_3, \alpha\}$. Then making use of Eqs. (46-48) we can obtain expressions of the quantum discord in different regimes of the initial state parameters. When $2|c_3| > |c_1 - c_2| + |c_1 + c_2|$, we have

$$\mathcal{D}(\hat{\rho}(t)) = 2 + \sum_{i=1}^4 \lambda_i \log_2 \lambda_i - \sum_{m=1}^2 \frac{\gamma_m}{2} \log_2 \gamma_m, \quad (49)$$

and when $2|c_3| < |c_1 - c_2|e^{-2|\alpha|^2} + |c_1 + c_2|$, we have

$$\mathcal{D}(\hat{\rho}(t)) = 2 + \sum_{i=1}^4 \lambda_i \log_2 \lambda_i - \sum_{m=1}^2 \frac{\gamma'_m}{2} \log_2 \gamma'_m, \quad (50)$$

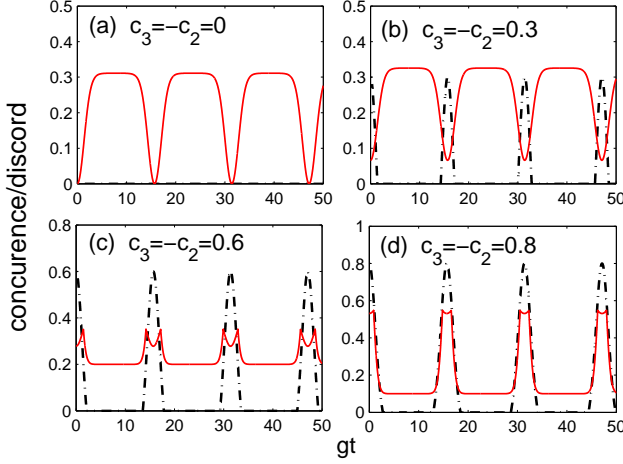


FIG. 2: (Color online) Time evolution of concurrence (dot dashed curves) and quantum discord (solid curves) for different values of c_3 when other parameters are taken by $c_1 = 1$, $c_2 = -c_3$, $\alpha = 2$, $g = 1$, and $\delta = 10$.

where two parameters γ_m and γ'_m are defined by

$$\gamma_m = 1 + (-1)^m |c_3|, \quad \gamma'_m = 1 + (-1)^m c_4. \quad (51)$$

In order to see clearly dynamic characteristics of the quantum entanglement and quantum discord, in the following we numerically investigate the time evolution of the concurrence given by Eqs. (29) and (30), the discord $\mathcal{D}(\hat{\rho}(t))$ given by Eqs. (49) and (50).

Fig. 2 indicates the influence of initial states of the two DQD spin qubits on dynamic evolution of the EC and DC when the DQD spin qubits and the TLR are initial X-type quantum states and a coherent state, respectively. We have plotted time evolution of concurrence (dot dashed curves) and quantum discord (solid curves) for some X-type initial quantum states of the two spin qubits when other parameters take fixed values. Fig. 2 reflects some interesting dynamic properties of EC and DC.

(i) Fig. 2 demonstrates that in the EC death regions there exist DC stationary states in which DC does not change in a finite time area. From Fig. 2(a-d) we can see that in the DC stationary states the stable DC amplification or degradation can be generated during the dynamical evolution. We can also see that these DC stationary states can be controlled by initial-state parameters, the coupling, and detuning between qubits and the TLR. Fig. 2(a-b) clearly indicate that the DC may be amplified in the most time of the death regime of the EC, and remains its maximal value. However, Fig. 2(c-d) reflect the fact that the DC may be degraded in the most time of the death regime of the EC, and remains its minimal value. These imply that one can generate stable DC amplification (degradation) in the EC death regions for certain initial X-type states.

(ii) Fig. 2 indicates the appearance of the full synchronization and anti-synchronization phenomena in the EC and DC time evolution, and shows that the EC and DC synchroniza-

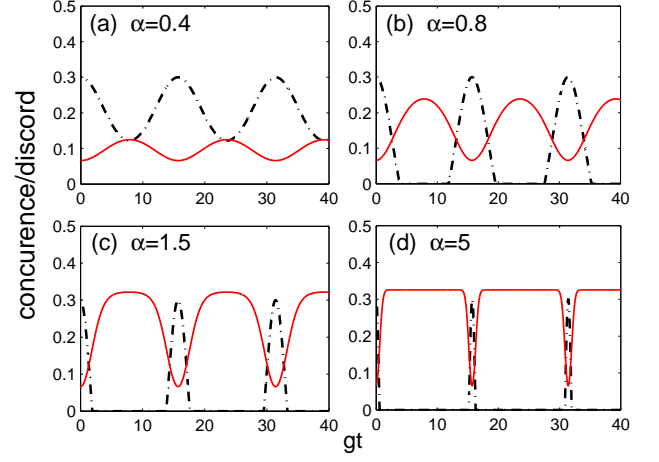


FIG. 3: (Color online) Time evolution of concurrence (dot dashed curve) and discord (solid curve) for different values of α . Other parameters are $c_1 = 1$, $c_2 = -c_3 = -0.3$, $g = 1$, and $\delta = 10$.

tion (anti-synchronization) depends on the initial-state parameters of the two DQD spin qubits. Fig. 2(c) and Fig. 2(d) correspond to the EC and DC synchronization evolution while Fig. 2(b) corresponds to the EC and DC anti-synchronization evolution. From Fig. 2 we can also see that both EC and DC time evolutions have the same period which is independent of c_2 and c_3 . Indeed, when $c_1 = 1$ and $1 > c_3 = -c_2 > 0$ the evolution period of EC and DC can be analytically obtained from Eqs. (28-30) with the simple expression $T = \delta\pi/(2g)$. This implies that the larger period can available by increasing (decreasing) the detuning δ (the effective coupling constant g).

(iii) Fig. 2 indicates that both EC and DC may exhibit sudden death phenomenon under certain conditions in their dynamic evolution. Fig. 2(a) shows that the discord sudden death (DSD) periodically occurs while EC always remains zero. From Fig. 2(b) we can see that the entanglement sudden death (ESD) periodically happens while DC almost remains its maximal value in the EC death regions.

In Fig. 3 we show the effect of initial states of the TLR on EC and DC dynamics. We have plotted time evolution of concurrence (dot dashed curves) and quantum discord (solid curves) for some initial quantum states of the TLR when other parameters take fixed values. From Fig. 3 we can see that the time evolutions of EC and DC are completely anti-synchronous with the same period, and the ESD occurs for enough large values of the initial parameter α in the EC dynamics. From Fig. 3(c) and Fig. 3(d) we can also see that the increase of the initial-state parameter α of the TLR not only leads to the ESD but also the appearance of the DC stationary states in the EC death regimes. The lifetime of these DC stationary states can be increased by increasing value of the initial-state parameter α .

In Fig. 4 we investigate the influence of the detuning parameter on dynamic evolution of the EC and DC. We have plotted time evolution of concurrence (dot dashed curves) and

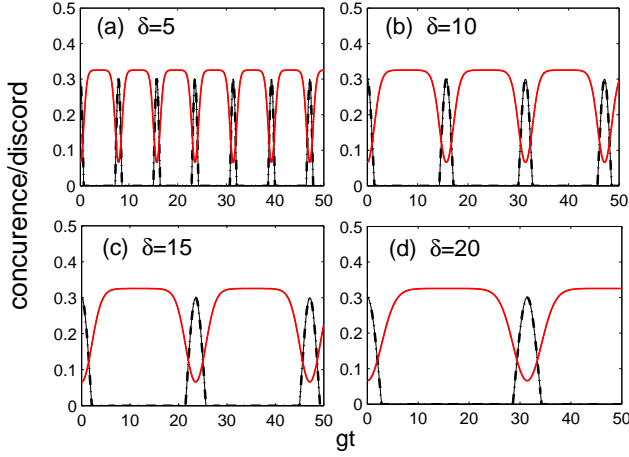


FIG. 4: (Color online) Time evolution of concurrence (dot dashed curves) and quantum discord (solid curves) for different values of δ when other parameters are taken by $c_1 = 1$, $c_2 = -c_3 = 0.3$, $\alpha = 2$, and $g = 1$.

quantum discord (solid curves) for different values of the detuning of the TLR for a fixed initial state of the combined system and a fixed coupling. From Fig. 4 we can see that the lifetime of the DC stationary states and the EC death regimes increases with the increase of the detuning parameter δ . The larger the detuning, the longer the lifetime of the DC stationary states and the EC death time become. By using Eqs. (28-30), similar numerical analysis indicate that the lifetime of the DC stationary states and the EC death regimes can be manipulated by changing the coupling constant g . Therefore, we can conclude that the lifetime of the DC stationary states and the EC death regimes can be controlled by changing the detuning parameter and the coupling constant between the DQD and the TLR.

IV. CONCLUDING REMARKS

In conclusion, we have proposed a magnetic coupling scheme between two DQD spin qubits by using the TLR as a bus system. We have shown that the inter-spin-qubit cou-

pling is controllable and switchable. The coupling controllability can be realized by changing external magnetic field. It is worthwhile to mention that the present scheme can be used to build a hybrid qubit system where DQD spin qubits are integrated together with superconducting qubits in the same TLR. We have studied in some detail dynamical behaviors of inter-qubit quantum correlations described by EC and DC when the two DQD spin qubits and the TLR are initially prepared in some X-type quantum states and a coherent state, respectively. We have studied the relation between EC and DC. We have demonstrated that in the EC death regions there exist DC stationary states. In these DC stationary states, the stable DC amplification or degradation can be generated during the dynamical evolution. We have shown that the lifetime of these DC stationary states depends on the initial-state parameters, the coupling, and detuning between qubits and the TLR. We have also found the full synchronization and anti-synchronization phenomena in the time evolution of the EC and DC, and indicated that the time synchronization (anti-synchronization) of the EC and DC dynamics depend on the initial-state parameters of the two DQD spin qubits. We have indicated that both EC and DC may exhibit sudden death phenomenon under certain conditions in their dynamic evolution, and the DC stationary states always appear in the EC death regions. The DC stationary states, synchronization and anti-synchronization found in the present paper highlight new characteristics in the EC and DC dynamics of the combined DQD-TLR system, and have potential applications in quantum information processing. It could be convinced that the magnetic coupling scheme suggested in the present paper can be used to realize quantum information processing since it takes advantages of controllable and switchable coupling and qubit scalability.

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